$$\Phi(z)=\frac{i2q\sin\alpha}{\sqrt{1+\gamma^2}}\left[\left(\frac{1}{\sqrt{1+\gamma^2}}-1\right)\left(\operatorname{arctg}\gamma z-\operatorname{arctg}\frac{\gamma\sqrt{z_1^2-1}}{\sqrt{1+\gamma^2}}\right)+\right.$$
$$\left.+\gamma\frac{\sqrt{1+\gamma^2}\,z_1-\sqrt{z_1^2-1}}{1+\gamma^2 z_1^2}\right],\quad z_1=\frac{1}{\gamma}\operatorname{tg}\frac{\pi z}{2a}. \tag{3.3}$$

In Fig. 1, the variation in the temperature drop $T^+ - T^-$ (referred to q) along the cracks is shown for $\alpha = \pi/2$ and b/a = 0.25 (1), 0.5 (2), 0.75 (3), 0.9 (4) for $0 \leqslant x/b \leqslant 1$.

NOTATION

T^+, T^-, values of temperature T at the left-hand and right-hand edges of the inclusions; ψ, current function; W, complex potential of temperature field; k_0, thermal conductivity of inclusions; k, thermal conductivity of body; Γ_n, smooth line in complex z plane; Γ, piecewise-continuous line ($\Gamma = \Gamma_1 + \ldots + \Gamma_N$); $2h_0h(s)$, width of inclusion in section s (h_0 = const); 2b, length of inclusion; $2a$, period of complex potential W.

LITERATURE CITED

1. I. M. Abdurakhmanov, Inzh.-Fiz. Zh., 22, No. 4 (1972).
2. M. A. Lavrent'ev and B. V. Shabat, Methods in the Theory of a Complex Variable [in Russian], Fizmatgiz, Moscow (1958).
3. L. G. Magnaradze, Soobshch. Akad. Nauk GruzSSR, 3, No. 6 (1942).

SOLVING A SET OF DIFFERENTIAL EQUATIONS OF HEAT AND ELECTRICAL TRANSFER

I. A. Zhvaniya, M. Z. Maksimov, and G. A. Tkhor

UDC 536.2.023

Methods are proposed in this article for solving the first boundary-value problem for a system of nonlinear differential equations for heat and electrical transfer in the general one-dimensional case.

1. It is known that the transfer of heat and charge in the media which possess thermoelectrical properties is governed by the equations of Maxwell, of heat conduction, and by the generalized Ohm's law. In the stationary case these equations can be written in the form [1, 2]

$$\operatorname{div}(\varkappa\nabla T)+\mathbf{JE}-\mathbf{J}\nabla(\alpha T)=0,$$
$$\mathbf{J}=\frac{1}{\rho}(\mathbf{E}-\alpha\nabla T),\ \operatorname{div}\mathbf{J}=0; \tag{1}$$

the solution of the equations under appropriate boundary conditions determines completely the fundamental characteristics of a thermoelement — the power generated W and the heat-flux density q:

$$W=-\int\mathbf{JE}\,dv,\ \mathbf{q}=-\varkappa\nabla T; \tag{2}$$

where T is temperature; E is the electric field intensity; $\alpha(T)$ is the coefficient of thermo-emf; $\varkappa(T)$ is the coefficient of thermal conductivity; and $\rho(T)$ is the coefficient of resistivity.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 3, pp. 516-523, March, 1977. Original article submitted July 23, 1975.

In a number of articles the determination of heat fluxes on the system boundaries has been considered in the case of given boundary temperatures [3-6]. For flat one-dimensional thermoelements approximate formulas have been obtained for the heat fluxes when the first-correction terms are retained and thermoelectrical effects reflected, though without a strong mathematical basis.

2. In the present article methods are proposed for solving the system (1) for the general one-dimensional case when the sought functions depend on $|\mathbf{x}| = x = \sqrt{\sum_{i=1}^{n} x_i^2}$, where n is the dimension of the space.

In our case the system (1) becomes

$$\frac{d}{dx}\left[x^{n-1}\varkappa(T)\frac{dT}{dx}\right] + J^2(x)\,x^{n-1}\rho(T) - J(x)\,x^{n-1}\tau(T)\frac{dT}{dx} = 0,$$

$$\frac{d}{dx}\left[x^{n-1}J(x)\right] = 0\ (x_0 < x < x_1),\ T(x_0) = T_0,\ T(x_1) = T_1,\ J(x_1) = J_1, \tag{3}$$

where $\tau(T) = T(d\alpha/dT)$ is the Thomson coefficient. Here and subsequently only the first boundary-value problem is considered.

Dimensionless quantities are introduced by means of

$$\xi = \frac{x}{x_1},\ y = \frac{T}{T_0},\ \Lambda(y) = \frac{\varkappa(T)}{\varkappa(T_0)},$$

$$\beta(y) = \frac{\tau(T)}{\alpha(T_0)},\ \gamma(y) = \frac{\rho(T)}{\rho(T_0)},\ a^2 = \frac{J_1^2\rho(T_0)x_1^2}{T_0\varkappa(T_0)},$$

$$\bar{Z} = \frac{\alpha^2(T_0)T_0}{\rho(T_0)\varkappa(T_0)} = Z(T_0)T_0 \tag{4}$$

as well as another independent variable,

$$\eta = \int_{\xi_0}^{\xi}\frac{dx}{x^{n-1}},\ \xi_0 = \frac{x_0}{x_1}, \tag{5}$$

which result in

$$\frac{d}{d\eta}\left[\Lambda(y)\frac{dy}{d\eta}\right] - a\sqrt{\bar{Z}}\,\beta(y)\frac{dy}{d\eta} + a^2\gamma(y) = 0,$$

$$y(0) = 1,\ y(\eta_1) = y_1 = \frac{T_1}{T_0}, \tag{6}$$

$$Q = -\Lambda\frac{dy}{d\eta} = \frac{q_x S(x)}{A}. \tag{7}$$

Here Q is the dimensionless flux; $S(x) = \pi 2^{n-1}x^{n-1}L^{3-n}$ is the area of current cross section; L is the length of the system in the perpendicular direction to the heat flow and current;

$$A = \pi 2^{n-1}x_1^{n-2}L^{3-n}\varkappa(T_0)T_0. \tag{8}$$

Equation (6) for the function $Q = Q(y)$ can be reduced to an Abel equation of the second kind whose explicit solution in terms of quadratures can be obtained for only special forms of $\Lambda(y)$, $\beta(y)$, and $\gamma(y)$; for example, when $\Lambda\gamma/\beta$ = const.

In particular, if $\gamma = \Lambda = 1$ and $\beta = \beta_0$ = const, one finds directly from (6)

$$y = 1 + \frac{a}{\sqrt{\bar{Z}}\beta_0}\eta + \frac{\exp[a\sqrt{\bar{Z}}\beta_0\eta] - 1}{\exp[a\sqrt{\bar{Z}}\beta_0\eta_1] - 1}\left[y_1 - 1 - \frac{a}{\sqrt{\bar{Z}}\beta_0}\eta_1\right], \tag{9}$$

$$Q = -\frac{a}{\sqrt{\bar{Z}}\beta_0} - \frac{a\sqrt{\bar{Z}}\beta_0\exp[a\sqrt{\bar{Z}}\beta_0\eta]}{\exp[a\sqrt{\bar{Z}}\beta_0\eta_1] - 1}\left[y_1 - 1 - \frac{a}{\sqrt{\bar{Z}}\beta_0}\eta_1\right], \tag{10}$$

from which familiar results [7-8] can be obtained for the case of n = 1 or 2.

It is of some importance to develop approximation methods to solve Eq. (6) without having to specify the functions $\Lambda(y)$, $\beta(y)$, and $\gamma(y)$. To this end the system (6) is now reduced to a nonlinear integral equation. If provisionally one regards Λ, β, and γ as known functions of η, one obtains for y

$$y = 1 - a^2 \int_0^{\eta} d\eta' \frac{1}{\Lambda} \int_0^{\eta'} d\eta'' \gamma \exp\left[-a\sqrt{\overline{Z}} \int_{\eta'}^{\eta''} \frac{\beta}{\Lambda} d\eta'''\right] +$$

$$+\frac{\int_0^{\eta} d\eta' \frac{1}{\Lambda} \exp\left[a\sqrt{\overline{Z}} \int_0^{\eta'} \frac{\beta}{\Lambda} d\eta''\right]}{\int_0^{\eta_1} d\eta' \frac{1}{\Lambda} \exp\left[a\sqrt{\overline{Z}} \int_0^{\eta'} \frac{\beta}{\Lambda} d\eta''\right]} \left[y_1 - 1 + a^2 \int_0^{\eta_1} d\eta \frac{1}{\Lambda} \int_0^{\eta} \gamma \exp\left[-a\sqrt{\overline{Z}} \int_{\eta}^{\eta'} \frac{\beta}{\Lambda} d\eta''\right] d\eta'\right] \tag{11}$$

and, correspondingly, for Q

$$Q = a^2 \int_0^{\eta} \gamma \exp\left[-a\sqrt{\overline{Z}} \int_{\eta}^{\eta'} \frac{\beta}{\Lambda} d\eta''\right] d\eta' - \frac{\exp\left[a\sqrt{\overline{Z}} \int_0^{\eta} \frac{\beta}{\Lambda} d\eta'\right]}{\int_0^{\eta_1} d\eta \exp\left[a\sqrt{\overline{Z}} \int_0^{\eta} \frac{\beta}{\Lambda} d\eta'\right]} \left[\int_1^{y'} \Lambda\, dy + a^2 \int_0^{\eta_1} d\eta \times\right.$$

$$\left.\times \int_0^{\eta} \gamma \exp\left[-a\sqrt{\overline{Z}} \int_{\eta}^{\eta'} \frac{\beta}{\Lambda} d\eta''\right] d\eta'\right]. \tag{12}$$

The above equations have the following advantages: one is able, on the one hand, to obtain from them directly the familiar results (9)-(10) for the properties remaining constant and, on the other hand, they are very convenient in the case of arbitrary $\Lambda(y)$, $\gamma(y)$, and $\beta(y)$ for finding the solution by any approximation method.

Indeed, Eq. (12) can be written in the form

$$Q = M(y, y_1; Q)\left[-a^2 L(y; Q) + \frac{M(y_1, 1; Q)}{M(y, 1; Q)}(1 + a^2 \overline{L(y; Q)})\right], \tag{13}$$

where

$$M(y', y''; Q) = \exp\left[a\sqrt{\overline{Z}} \int_{y'}^{y''} \frac{\beta}{Q} dy\right], \quad L(y; Q) =$$

$$= \int_1^{y} \frac{\Lambda\gamma}{Q} M(y_1, y'; Q)\, dy'; \quad \overline{f(y; Q)} = \frac{\int_1^{y'} \frac{\Lambda}{Q} f(y; Q)\, dy}{\int_1^{y_1} \Lambda dy}, \tag{14}$$

which represents a functional equation for the unknown Q(y). To find the solution one can use here the method of successive approximations. If for the zeroth approximation one adopts the solution of our problem in which the thermoelectrical effects are ignored (a, $\overline{Z} \to 0$), that is,

$$Q_0 = -\frac{1}{\eta_1} \int_1^{y_1} \Lambda dy, \tag{15}$$

then the k-th approximation is obtained from (13) and is equal to

$$Q_k = M(y, y_1; Q_{k-1})\left[-a^2 L(y; Q_{k-1}) + \frac{M(y_1, 1; Q_{k-1})}{M(y, 1; Q_{k-1})}[1 + a^2 \overline{L(y; Q_{k-1})}]\right]. \tag{16}$$

Since each approximation satisfies the boundary conditions of the problem, the proposed iteration procedure must converge to the true solution.

Since the flux Q is known, it is easy to obtain the sought temperature field in a quadrature:

$$\eta = -\int_1^y \frac{\Lambda dy'}{Q}. \tag{17}$$

Thus, the method enables one to obtain the solution of the problem under investigation with any degree of accuracy.

3. If the dimensionless parameter satisfies the inequality $a < 1$, then the solution can also be sought in the form of a power series:

$$y(\eta, \bar{Z}, a) = \sum_{i=0}^{\infty} a^i u_i(\eta, \bar{Z}). \tag{18}$$

By substituting this expansion in (6) and setting the coefficients of different powers of a equal to zero, a system of differential equations is obtained for the expansion coefficients $u_i(\eta, \bar{Z})$, $u_0(\eta, \bar{Z})$ satisfying at the same time the original boundary conditions (6) while $u_i(\eta, \bar{Z})$, $i \geqslant 1$ is equal to zero:

$$u_i(0, \bar{Z}) = u_i(\eta_1, \bar{Z}) = 0. \tag{19}$$

Since the solution for the zeroth approximation is known, it is not difficult to write them down for $i \geqslant 1$. The expressions are now given for the first three coefficients:

$$\Lambda(u_0)\frac{du_0}{d\eta} = \frac{G(y_1)}{\eta_1},$$

$$u_1 = \frac{\sqrt{\bar{Z}}\, G(u_0)\,\eta_1}{\Lambda(u_0)\, G(y_1)}\left[\overline{P(u_0)} - \overline{P(y_1)}\right], \tag{20}$$

$$u_2 = -\frac{1}{2}u_1^2\frac{d\ln\Lambda(u_0)}{du_0} + \frac{G(u_0)\,\eta_1}{\Lambda(u_0)\, G(y_1)}\left[\overline{\Phi(u_0)} - \overline{\Phi(y_1)}\right],$$

where $G(y) = \int_1^y \Lambda(x)dx$ is the Kirchhoff function;

$$\Phi(y) = \sqrt{\bar{Z}}\,\beta u_1(y) - g(y);\quad P(y) = \int_1^y \beta(x)\,dx;$$

$$g(y) = \frac{\eta_1}{G(y_1)}\int_1^y \gamma(x)\,\Lambda(x)\,dx;\quad \overline{f(y)} = \frac{\int_1^y f(x)\,\Lambda(x)\,dx}{\int_1^y \Lambda(x)\,dx}. \tag{21}$$

Knowing the temperature distribution, one can now find the heat flux:

$$-\Lambda\frac{dy}{d\eta} = \Theta_0 + a\Theta_1 + a^2\Theta_2 + 0(a^2), \tag{22}$$

$$\Theta_0 = -\Lambda(u_0)\frac{du_0}{d\eta} = -\frac{G(y_1)}{\eta_1} = Q_0,\quad \Theta_1 = -\frac{d}{d\eta}\left[\Lambda(u_0)\,u_1\right] =$$

$$= -\sqrt{\bar{Z}}\left[P(u_0) - \overline{P(y_1)}\right], \tag{23}$$

$$\Theta_2 = -\frac{d}{d\eta}\left[\Lambda(u_0)\,u_2 + \frac{1}{2}u_1^2\frac{d\Lambda(u_0)}{du_0}\right] = -\Phi(u_0) + \overline{\Phi(y_1)}.$$

It should be mentioned here that in [4] similar solutions are given for the flux with an accuracy up to quadratic terms in the current (a^2); however, in the expression for Θ a term which is quadratic in the Thomson effect was omitted.

For the sake of comparison, the expression is now given for the heat flux with an accuracy up to a^2 in the case of the properties of the material remaining unchanged ($\Lambda = \gamma = 1$, $\beta = \beta_0 = \text{const}$). By using the solution (10), one obtains

$$Q = \frac{1-y_1}{\eta_1} + a\sqrt{\bar{Z}}\,\beta_0(1-y_1)\frac{2\eta - \eta_1}{2\eta_1} + \frac{a^2}{2}(2\eta - \eta_1) +$$

$$+\frac{a^2\bar{Z}}{12}\eta_1\beta_0^2(1-y_1)+\frac{a^2\bar{Z}\beta_0^2}{2\eta_1}(1-y_1)(\eta-\eta_1)\eta. \quad (24)$$

It can easily be seen that formula (22) goes into (24) in the case of the properties remaining constant.

4. Moreover, the solution of system (6) can also be represented by a power series in the powers of η. Our attention is now turned to the Kirchhoff function; the system (6) now becomes

$$\frac{d^2G}{d\eta^2}-\frac{a\sqrt{\bar{Z}}\beta[G]}{\Lambda[G]}\cdot\frac{dG}{d\eta}+a^2\gamma[G]=0,$$

$$G(0)=0,\ G(\eta_1)=\int_1^{y_1}\Lambda(y)\,dy=G_1. \quad (25)$$

Its solution is sought in the form of the following series:

$$G(\eta)=\sum_{n=0}^{\infty}G^{(n)}(0)\frac{\eta^n}{n!}, \quad (26)$$

where the expansion coefficients $G^{(n)}(0)$ can be determined by using the boundary conditions and the main equation.

It is not possible to write down explicitly the series (26) in the general case; however, by retaining only the first three expansion terms one obtains

$$G(\eta)=A\eta+B\eta^2, \quad (27)$$

where

$$A=\frac{2G_1+\eta_1^2a^2\gamma(1)}{2\eta_1+\eta_1^2\dfrac{a\sqrt{\bar{Z}}\beta(1)}{\Lambda(1)}};\ B=A\frac{a\sqrt{\bar{Z}}\beta(1)}{2\Lambda(1)}-\frac{a^2\gamma(1)}{2}, \quad (28)$$

and in this case the heat flux is equal to

$$Q=-\frac{dG}{d\eta}=-A-2B\eta. \quad (29)$$

For $a<1$ this expression with an accuracy up to $\sim a^2$ becomes

$$Q=-\frac{G_1}{\eta_1}-\frac{G_1}{\eta_1}\frac{a\sqrt{\bar{Z}}\beta(1)}{\Lambda(1)}\cdot\frac{2\eta-\eta_1}{2\eta_1}+a^2\gamma(1)\frac{2\eta-\eta_1}{2}-G_1\frac{\eta_1a^2\bar{Z}\beta^2(1)}{4\Lambda^2(1)}. \quad (30)$$

The formulas (28)-(30) enable us to obtain an explicit relation for the heat flux in terms of the coordinate η.

5. Various methods have been considered above for solving the system of equations (6)-(7), and several approximate relations have been obtained for evaluating the temperature and heat fluxes in one-dimensional thermoelements.

We now proceed to the evaluation of power. It follows from the relations (1)-(2) that

$$W=I\int_{T_1}^{T_0}\alpha(T)\,dT-I^2R_{\text{int}}, \quad (31)$$

where $I=j(x)S(x)$ is the total current in the system and R_{int} is the internal resistance of the thermoelement, which for one-dimensional systems is given by

$$R_{\text{int}}=\int_{x_0}^{x_1}\rho[x]\frac{dx}{S(x)}=b\int_1^{y_1}\frac{\gamma(y)\Lambda(y)}{Q(y)}dy, \quad (32)$$

where

$$b=-\frac{\rho(T_0)}{\pi 2^{n-1}L^{3-n}x_1^{n-2}}, \quad (33)$$

that is, the internal resistance depends not only on the geometry of the element, but also on the temperature distribution.

Approximate formulas are now found to compute R_{int} by employing the previously obtained results for the heat flux.

By restricting our considerations to the first approximation, one obtains from (16)

$$Q = M_0(y, y_1)\left[-a^2 L_0(y) + \frac{M_0(y_1, 1)}{M_0(y, 1)}(1 + a^2 \overline{L_0(y)})\right], \tag{34}$$

where

$$M_0(y', y'') = \exp\left[\frac{-a\sqrt{Z}}{Q_0}\int_{y'}^{y''}\beta dy\right];\ L_0(y) = -\frac{\eta_1}{G_1}\int_1^y \gamma\Lambda M_0(y_1, y')\,dy;\ \overline{f(y)} = -\frac{\eta_1}{G_1^2}\int_1^{y_1}\Lambda f(y)\,dy; \tag{35}$$

by substituting (34) into (32) and integrating, one obtains the following formula for the internal resistance:

$$R_{\text{int}} = -\frac{bQ_0}{a^2}\ln\left[1 - \frac{a^2 L_0(y_1)\overline{M_0(y, 1)}}{M_0(y_1, 1)(1 + a^2\overline{L_0(y)})}\right], \tag{36}$$

which can be used to obtain the volt—ampere characteristic of the thermoelements

$$V = \int_{T_1}^{T_0}\alpha(T)\,dT - IR_{\text{int}} \tag{37}$$

(V is voltage) and to estimate its deviation from the straight line.

For small $a < 1$, by employing the expansions (18) and (21) and the formula (22), one obtains the following expressions to determine the internal resistance:

$$R_{\text{int}} = \frac{b}{Q_0}\int_1^{y_1}\gamma\Lambda\left[1 - a\frac{\Theta_1}{Q_0}\right]dy = \frac{b}{Q_0}\int_1^{y_1}\gamma\Lambda\left[1 + au_1\frac{d\ln\gamma}{dy}\right]dy. \tag{38}$$

The function $(|u_1|)$ vanishes at the integration limits [see (19)]; therefore, it reaches its maximum inside this interval. By evaluating the correction to the resistance by the method of steepest descent, that is, taking the slowly varying function $u_1\Lambda(\partial\gamma/\partial y)/(y - 1)\cdot(y - y_1)$ outside the integral sign at the maximum point for the function $(y - y_1)(y - 1)$, $y^* = (1 + y_1)/2$, one obtains

$$R_{\text{int}} = \bar{R} - \frac{2ba}{3Q_0}(1 - y_1)\left[u_1\Lambda\frac{\partial\gamma}{\partial y}\right]_{y=y^*}. \tag{39}$$

To find improved estimates of the correction one can use the expansions

$$\Lambda(y) = \Lambda(y^*) + \Lambda'(y^*)(y - y^*) + \ldots,\ \beta(y) = \beta(y^*) + \beta'(y^*)(y - y^*) + \ldots$$

and by retaining the principal terms only one finally obtains

$$R_{\text{int}} = \bar{R} - \frac{b(1 - y_1)\sqrt{Z}a\eta_1^2}{12}\left[\frac{\beta}{\Lambda}\frac{\partial\gamma}{\partial y}\right]_{y=y^*}, \tag{40}$$

where

$$\bar{R} = \frac{b}{Q_0}\int_1^{y_1}\gamma\Lambda dy.$$

If one employs the solution (29), one can obtain the following formula for the internal resistance:

$$R_{\text{int}} = b\int_0^{G_1}\frac{\gamma[G]\,dG}{\sqrt{B^2 + 4AG}}. \tag{41}$$

By employing this formula one can easily find an explicit formula for R_{int} in terms of the system parameters if $\gamma[G]$ is known.

6. Thus, in the present article methods of solution have been given for the system (6)-(7) which enable one to find temperature fields and fluxes.

With the aid of these methods, approximation formulas have been found for the main characteristics of a thermoelement: the heat flux and the internal resistance.

It has been shown that in the previously proposed formulas for the heat flux a quadratic (in the Thomson effect) term is missing, and in the resistance a linear (in the current term) is also missing. The estimates show that these effects may result in the deviation of the volt—ampere characteristics from linear of several percent.

NOTATION

T, y, u, temperatures; q, Q, Θ, heat fluxes; x, ξ, η, coordinates; ϰ, Λ, thermal conductivities; ρ, γ, resistivities; α, coefficient of thermo-emf; τ, β, Thomson coefficients; L, length; S, cross section; J, current density; I, total current; E, electric field intensity; W, power; V, voltage.

LITERATURE CITED

1. L. D. Landau and E. M. Lifshits, Electrodynamics of Continuous Media, Addison-Wesley (1960).
2. A. G. Samoilovich and L. L. Korenblit, UFI, 10, 9, No. 2, 243 (1953).†
3. A. I. Burshtein, Physical Basis for Evaluation of Semiconductor Thermoelectric Devices [in Russian], Fizmatgiz, Moscow (1962).
4. B. Ya. Moizhes, Fiz. Tverd. Tela, 2, No. 4 (1960).
5. L. S. Okhotin, A. A. Efremov, V. S. Okhotin, and A. S. Pushkarskii, Thermoelectrical Generators [in Russian], Atomizdat, Moscow (1971).
6. S. Friedman, in: Direct Energy Transformation [Russian translation], Mir, Moscow (1969).
7. A. F. Ioffe, L. S. Stil'bans, E. K. Iordanishvili, and T. S. Stavitskaya, Thermoelectrical Cooling [in Russian], Izd. Akad. Nauk SSSR, Moscow (1956).
8. Yu. I. Tsvetkov, Izv. Vuzov, Énergetika, No. 4 (1974).

†As in Russian original.

APPLICATION OF INFINITE SYSTEMS TO THE SOLUTION OF BOUNDARY-VALUE PROBLEMS OF STEADY THERMAL CONDUCTION IN NONUNIFORM MEDIA

Yu. I. Malov and L. K. Martinson

UDC 536.24

A method of calculating the temperature field in nonuniform media is described. Examples of the calculation of the temperature distribution for an exponential variation of the thermal conductivity of the medium and also in a multilayer structure are presented.

In the rectangular region $\Omega\{0 \leqslant x \leqslant l,\ 0 \leqslant y \leqslant 1\}$ we will consider the boundary-value problem of steady thermal conduction [1]

$$\frac{\partial}{\partial x}\left[h(x)\ \frac{\partial u}{\partial x}\right]+h(x)\frac{\partial^2 u}{\partial y^2}-q^2u=-f(x,\ y), \tag{1}$$

$$u=0 \text{ on } \partial\Omega, \tag{2}$$

N. E. Bauman Moscow Higher Technical School. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 3, pp. 524-532, March, 1977. Original article submitted March 18, 1976.